Even better fixed-parameter algorithms for bicluster editing

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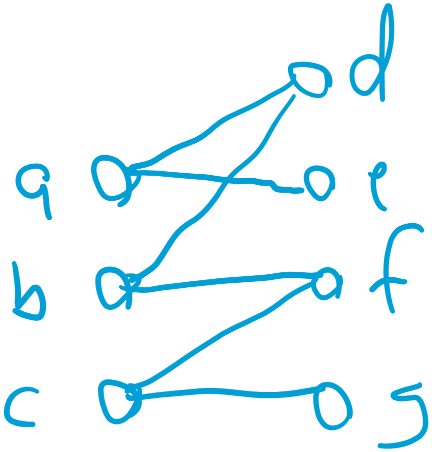
Bicluster editing

- **<u>Given</u>**: bipartite graph $G = (V_1 \cup V_2, E)$
- **<u>Goal</u>** : add/remove a minimum number of edges so that every connected component is a bicluster (a complete bipartite graph).



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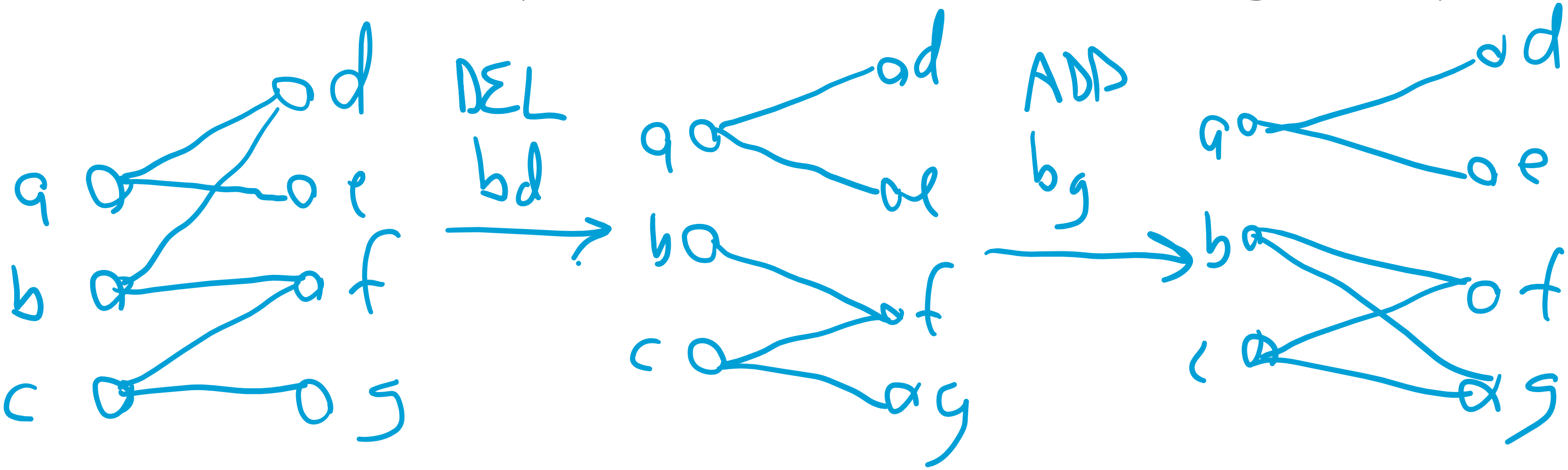

Some applications

- Gene expression analysis
- Social network analysis
- Phylogenetic analysis

Previous work

- NP-hard on subcubic graphs [Drange & al, 2015]
  - (actually ETH-hard)
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- $O^*(3.24^k)$ time FPT algorithm [Guo & al., 2008]
  - Kernel of $4k$ vertices (inaccuracy)
  - More likely $6k$, but remains to be proved

In this work

- $O^*(2.695^k)$ time FPT algorithm
  - Simple branching algorithm
- Kernel of $5k$ vertices

$O^*(2.695^k)$ time algorithm

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  - $N(u) \cap N(v) \neq \emptyset$
  - $N(u) \neq N(v)$
- Either u, v in the same bicluster, or not.
  - Both choices induce some cost.

$O^*(2.695^k)$ time algorithm

- Put u, v in same bicluster
  - Each node in $N(u) \Delta N(v)$ requires an insertion or deletion.
  - $2^{|N(u) \Delta N(v)|}$ ways, each requiring $|N(u) \Delta N(v)|$ edits.

$O^*(2.695^k)$ time algorithm

- Put u, v in different bicluster
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```
function biclusterize(G, k)
    if k < 0 then  Report "NO" and return
    Remove from G all bicluster connected components (Rule 1)
    if G has no vertex then  Report "YES" and return
    if G has maximum degree 2 then  Solve G in polynomial time

    Let u, v ∈ V(G) such that N(u) ∩ N(v) ≠ ∅ and N(u)△N(v) ≠ ∅
    Let R_u be the twin class of u and R_v be the twin class of v
    /*Put u, v in the same bicluster*/
    for each subset Z of N(u)△N(v) do
        Obtain G' from G by:
         inserting all missing edges between R_u ∪ R_v and Z and
         deleting all edges between R_u ∪ R_v and (N(u)△N(v)) \ Z
        Let h be the number of edges modified from G to G'
        biclusterize(G', k − h)
    end
    /*Put u, v in different biclusters*/
    for each subset Z of N(u) ∩ N(v) do
        Obtain G' from G by:
         deleting all edges between R_u and (N(u) ∩ N(v)) \ Z and
         deleting all edges between R_v and Z
        Let h be the number of edges modified from G to G'
        biclusterize(G', k − h)
    end
    if some recursive call reported "YES" then report "YES"
    else report "NO"
```

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- Branching recurrence

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- branching factor 3.237 => $O^*(3.237^k)$ time algorithm

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- Actually the same as in [Guo & al., 2008]

- **Theorem**: if G has a vertex of degree 1, then one can attain branching factor 2.066.

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  - Assuming it uses degree 1 special branching if a degree 1 vertex apears in a recursion.

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- => $O^*(2.695^k)$ time algorithm

Theorem: BICLUSTER EDITING admits a kernel of size 5k.

Kernel of size $5k$

- **Rule 1** : if a connected component X of G is a bicluster, remove X from G.

Kernel of size $5k$

- **Rule 2** : if there is a set of twins R such that $|R| > |N\big(N(R)\big)\backslash R|$, remove any vertex from R.
    - twins = on same side, have same neighbors

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  - Claim : imply $4k$ kernel
  - False : P_6 cannot be reduced

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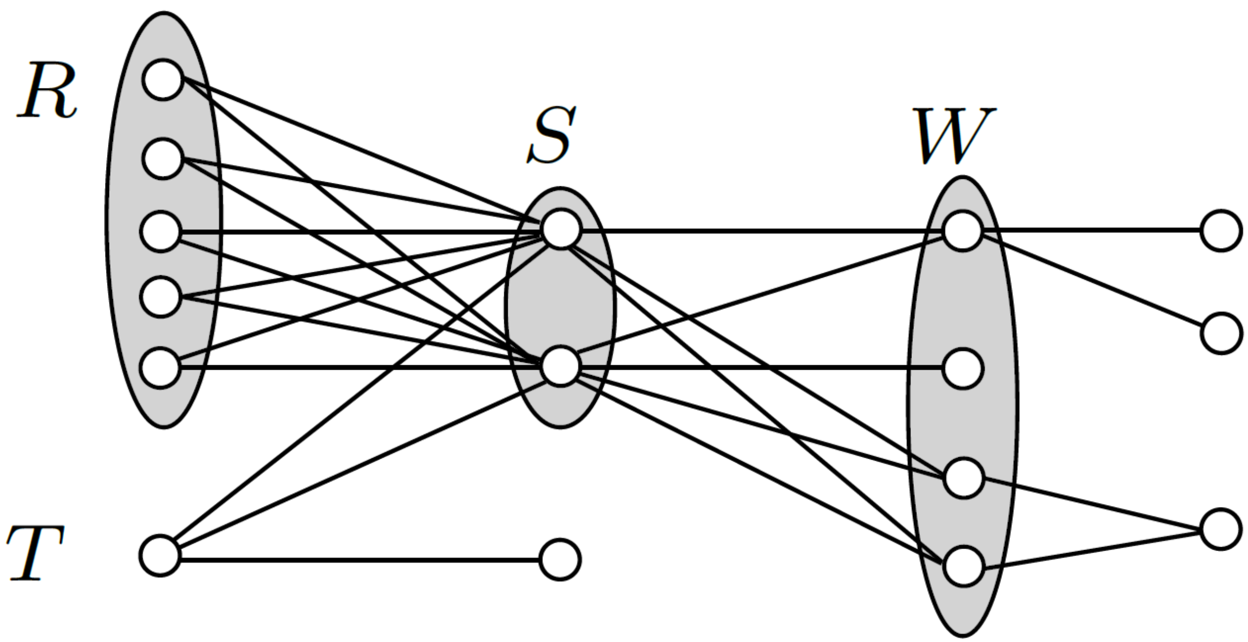

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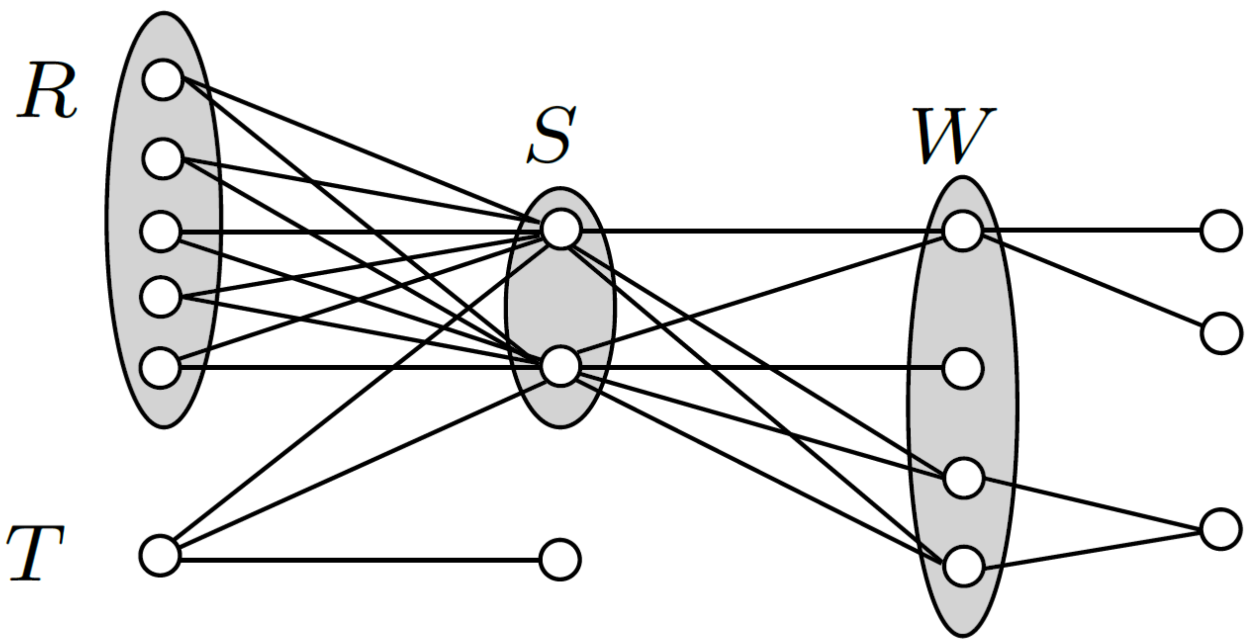
- **Rule 3** : let R be twins and T its sisters.
  If $|R| > |N(N(R)) \setminus \{R \cup T\}|$, then remove any edge from $t \in T$ to its neighbor outside $N(R)$.



Kernel of size $5k$

- **Thm**: Rules 1,2,3 are safe and lead to a kernel of size $5k$.
- Read paper for proof idea.

- Open: true kernel size of Rules 1,2,3?

Conclusion

- Better analysis = better than $O^*(2.695^k)$?
- Better analysis = better than $5k$ kernel?

- Ideas applicable to Cluster editing?
- Existence of $O^*(2^k)$ algorithm or better?