HOW BROKERS CAN OPTIMALLY ABUSE TRADERS

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GameStop (GME)

In foreign exchanges

- when you **buy**, the broker **sells**
- When you **sell**, the broker **buys**

EUR/USD chart

Bitcoin/USD

Market Summary > Bitcoin

31,274.90 USD

-4,371.50 (12.26%) ↓ past year

May 30, 21:00 UTC · Disclaimer

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max

NextAdvisor
In Partnership with TIME

How Crypto Investors Can Avoid the Scam That Captured $2.8 Billion in 2021

CNBC

Robinhood restricts trading in GameStop, other names involved in frenzy

BY ETORO

Where do you think the Brokers EARN their Profits from?

Broker conspiracies

- Buying or selling assets → Brokers handle transaction
- Have access to all open trades
- Large institutions (possibly) able to manipulate prices

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  - Assume total control over prices
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- **Goal**: how to help evil brokers as much as possible.
  - Assume total control over prices
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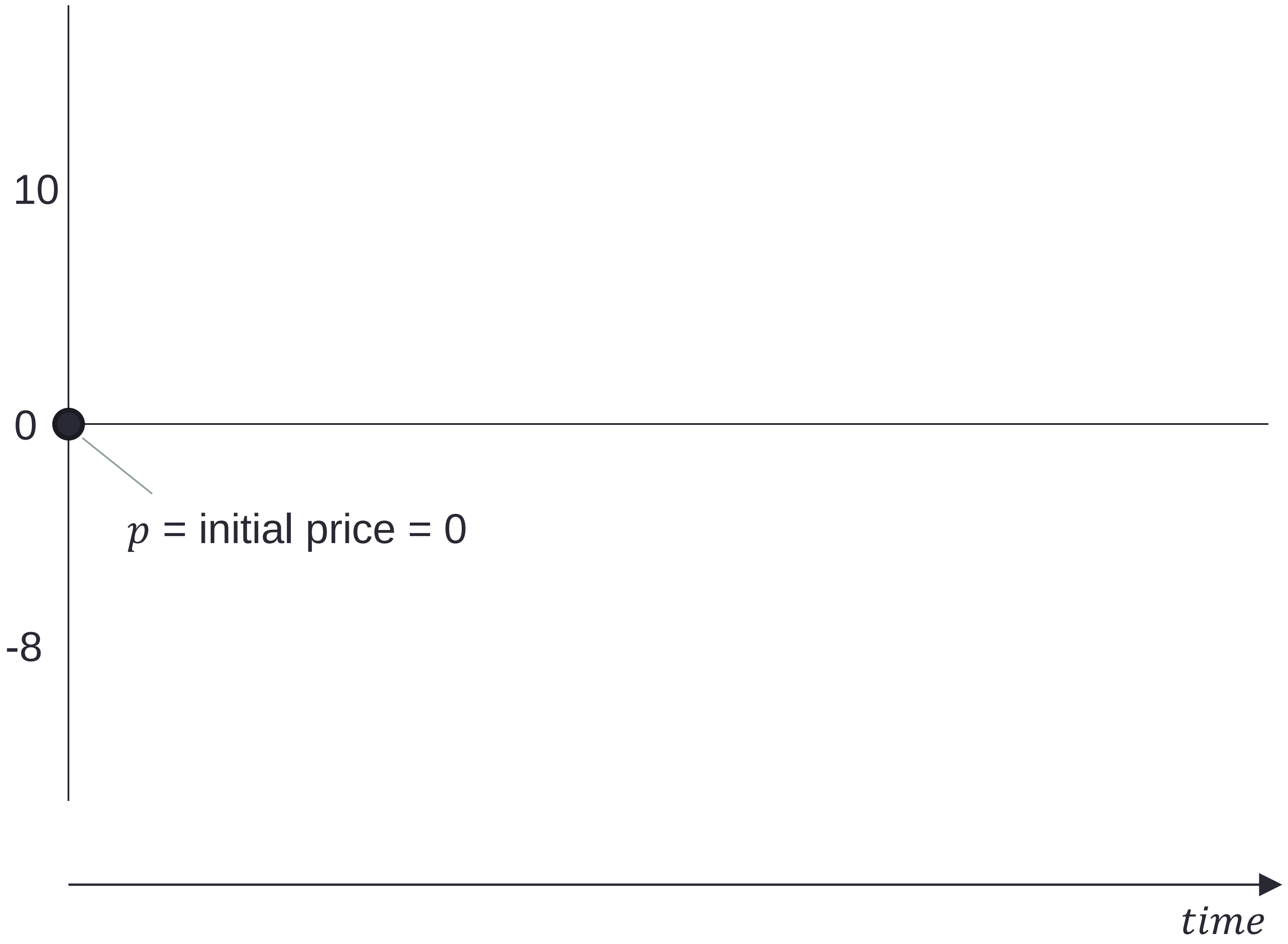
- **How is that FUN?!**
  - **No quant finance. That's boring! (at least for me)**
  - **Only combinatorial problems.**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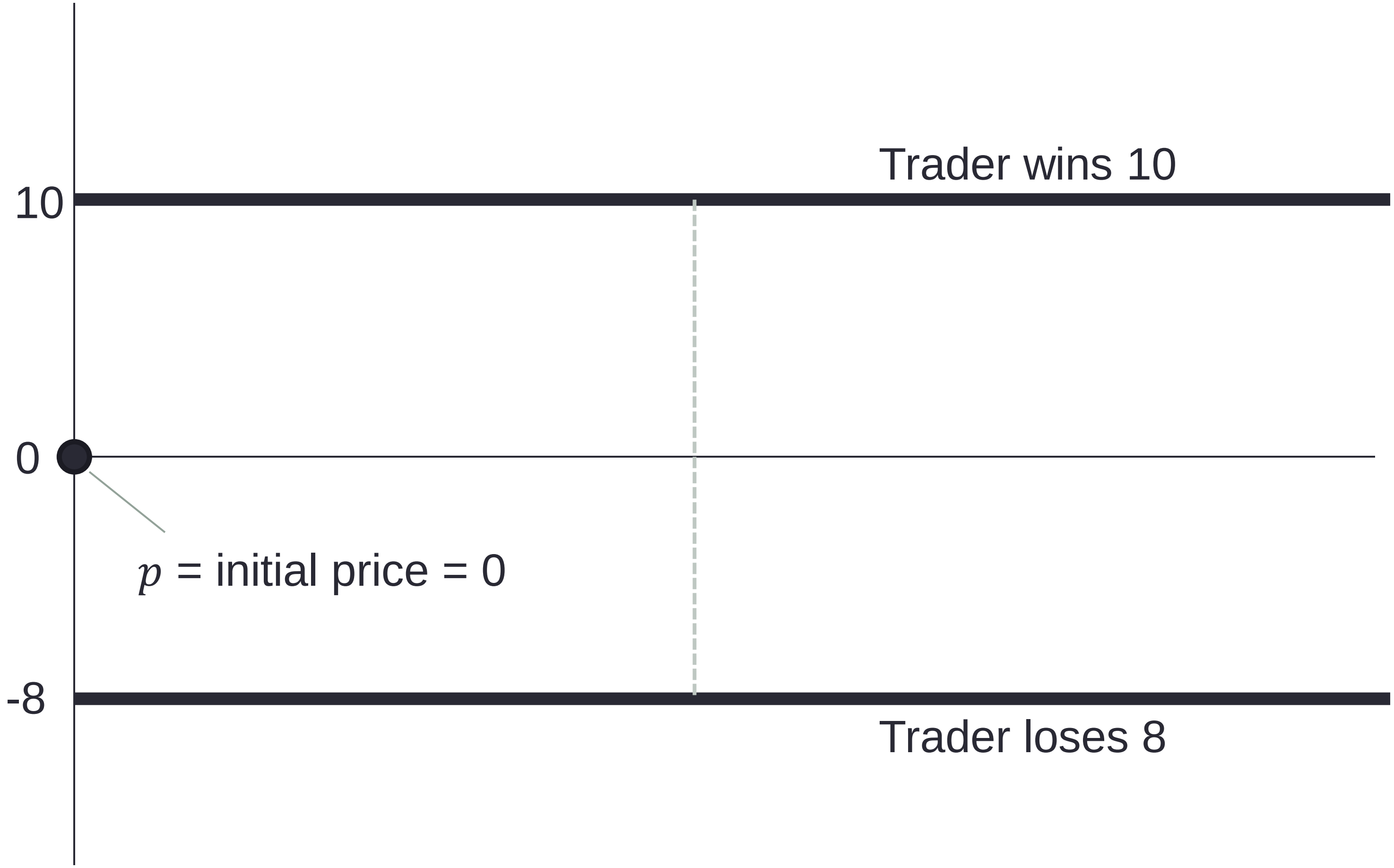
Trader wins 10
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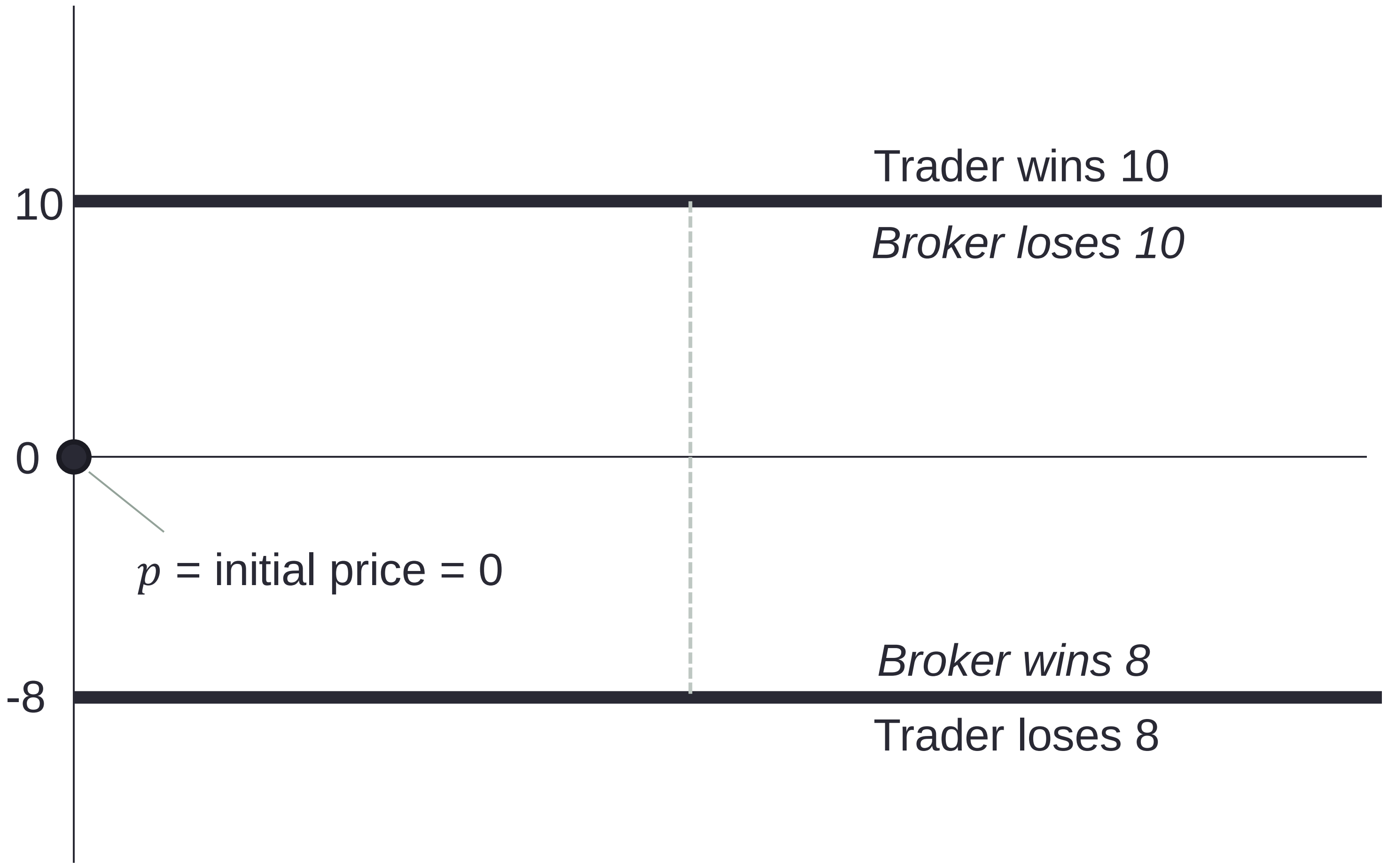
Trader loses 8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Broker loses 10
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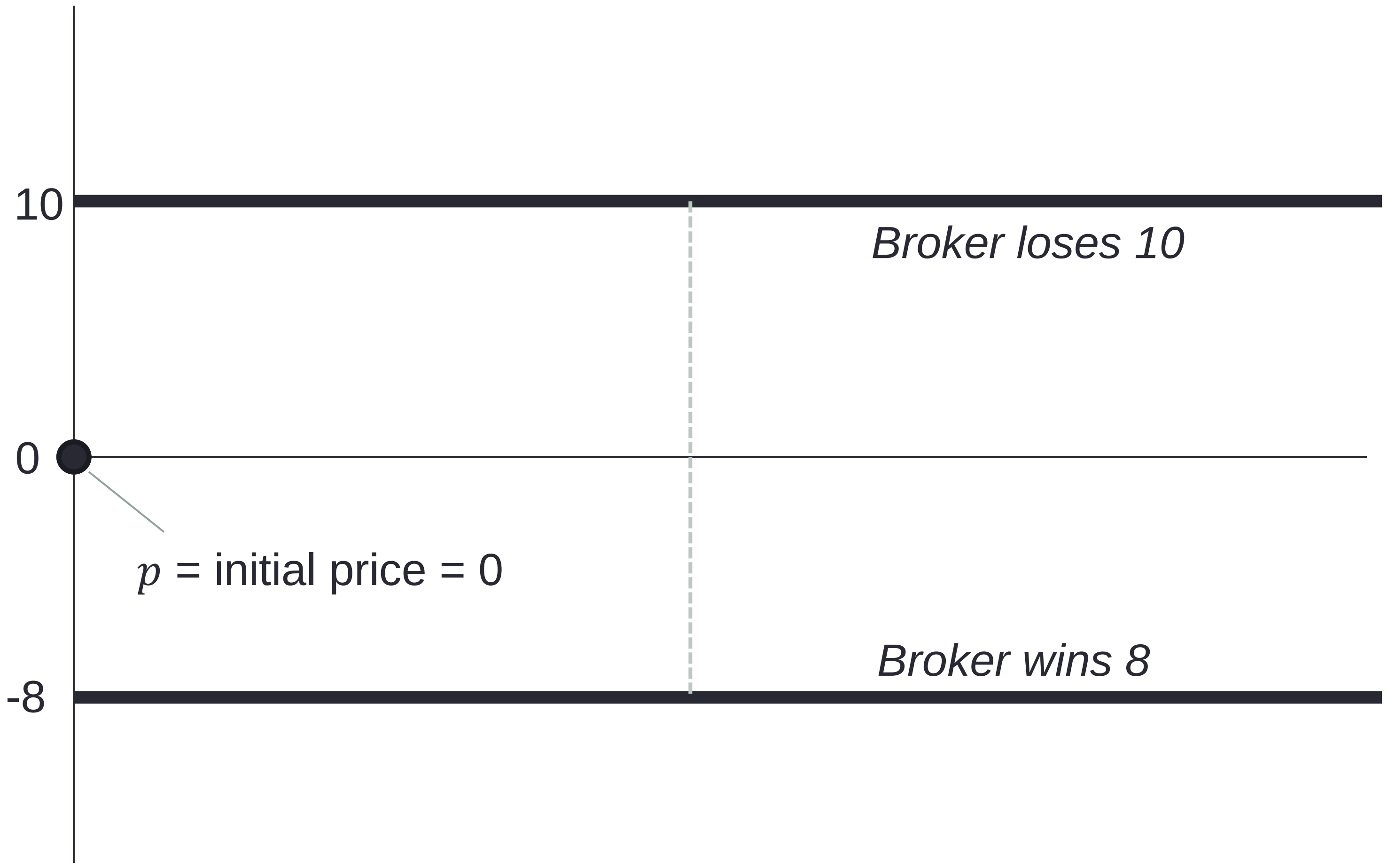
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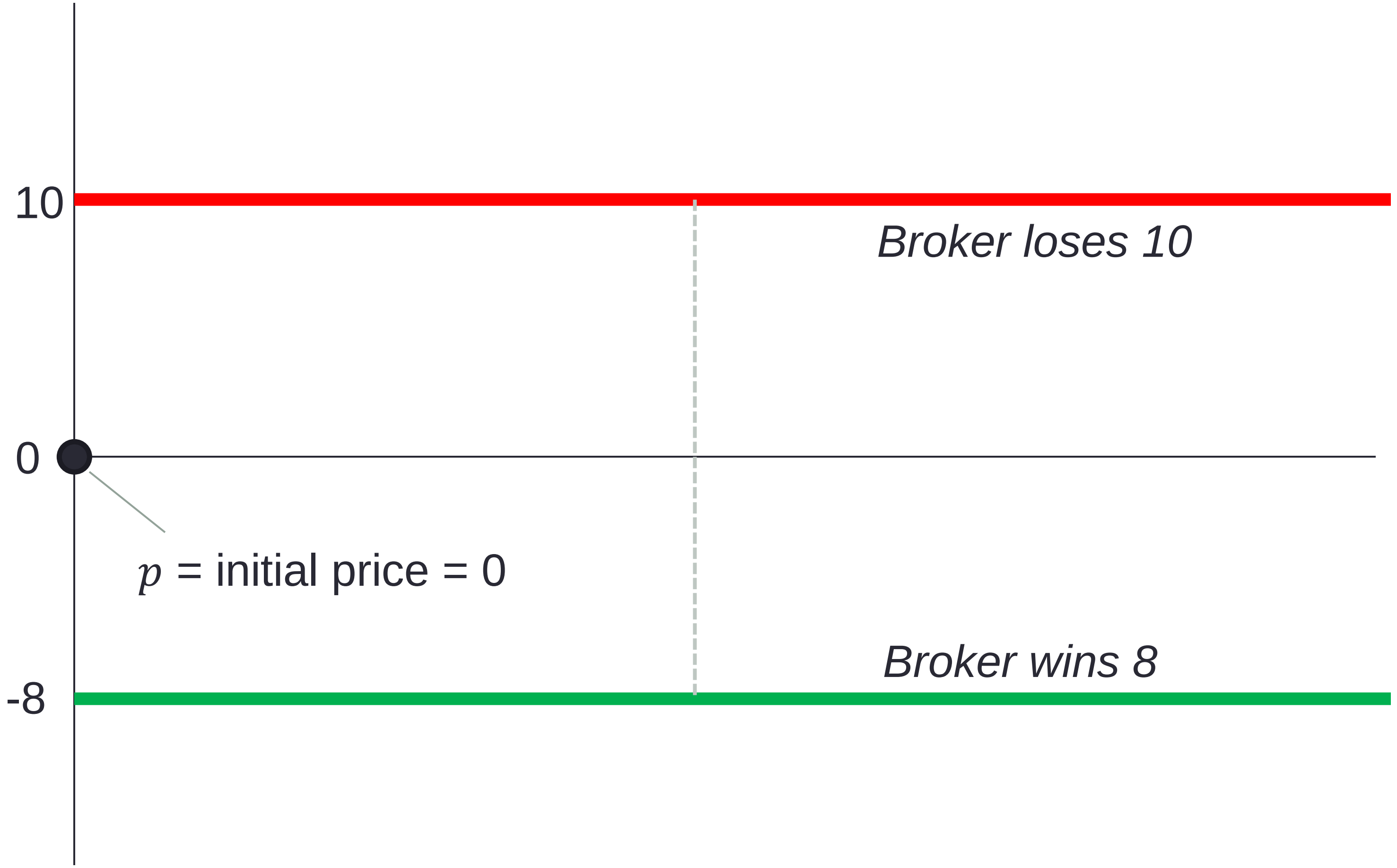
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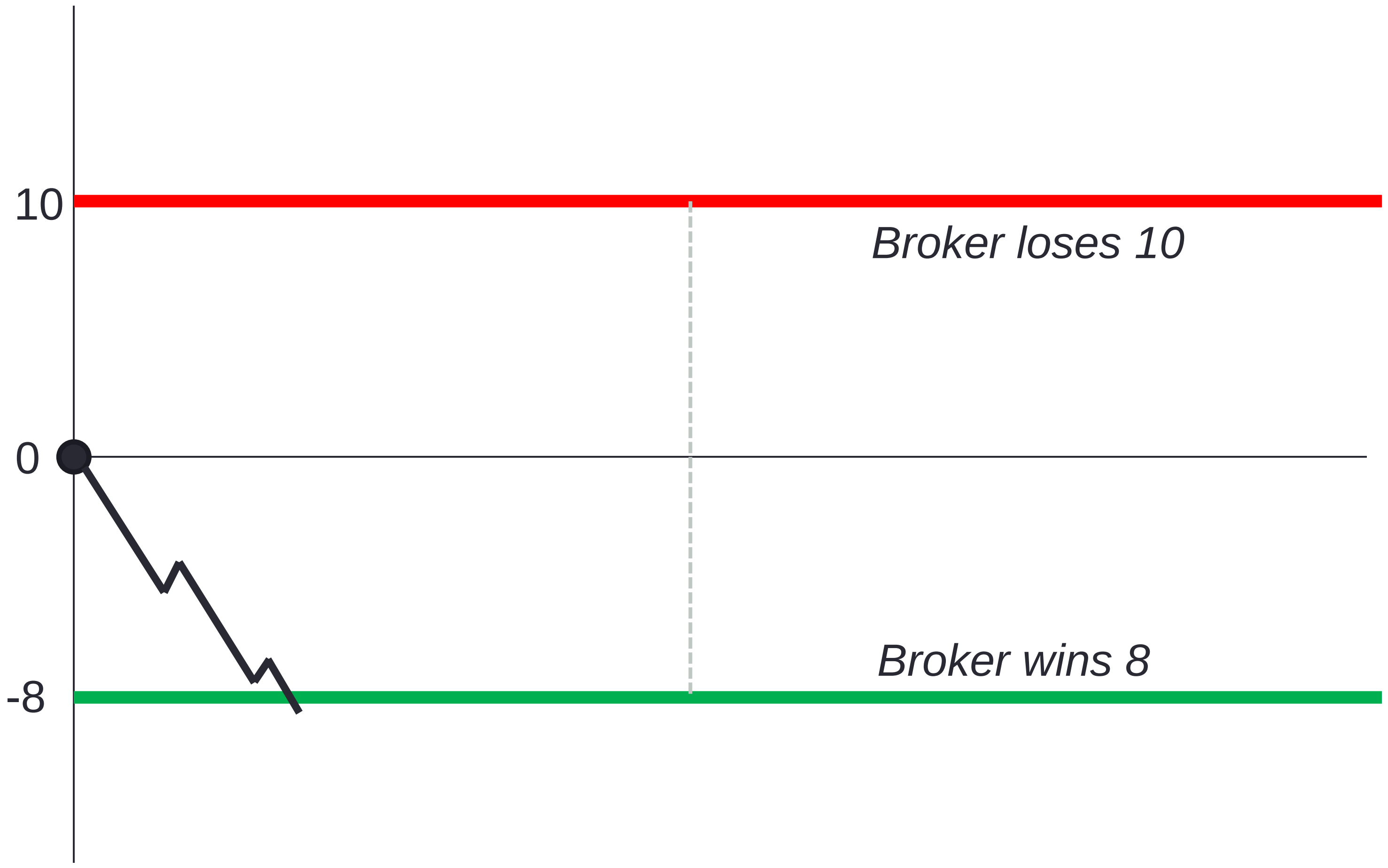
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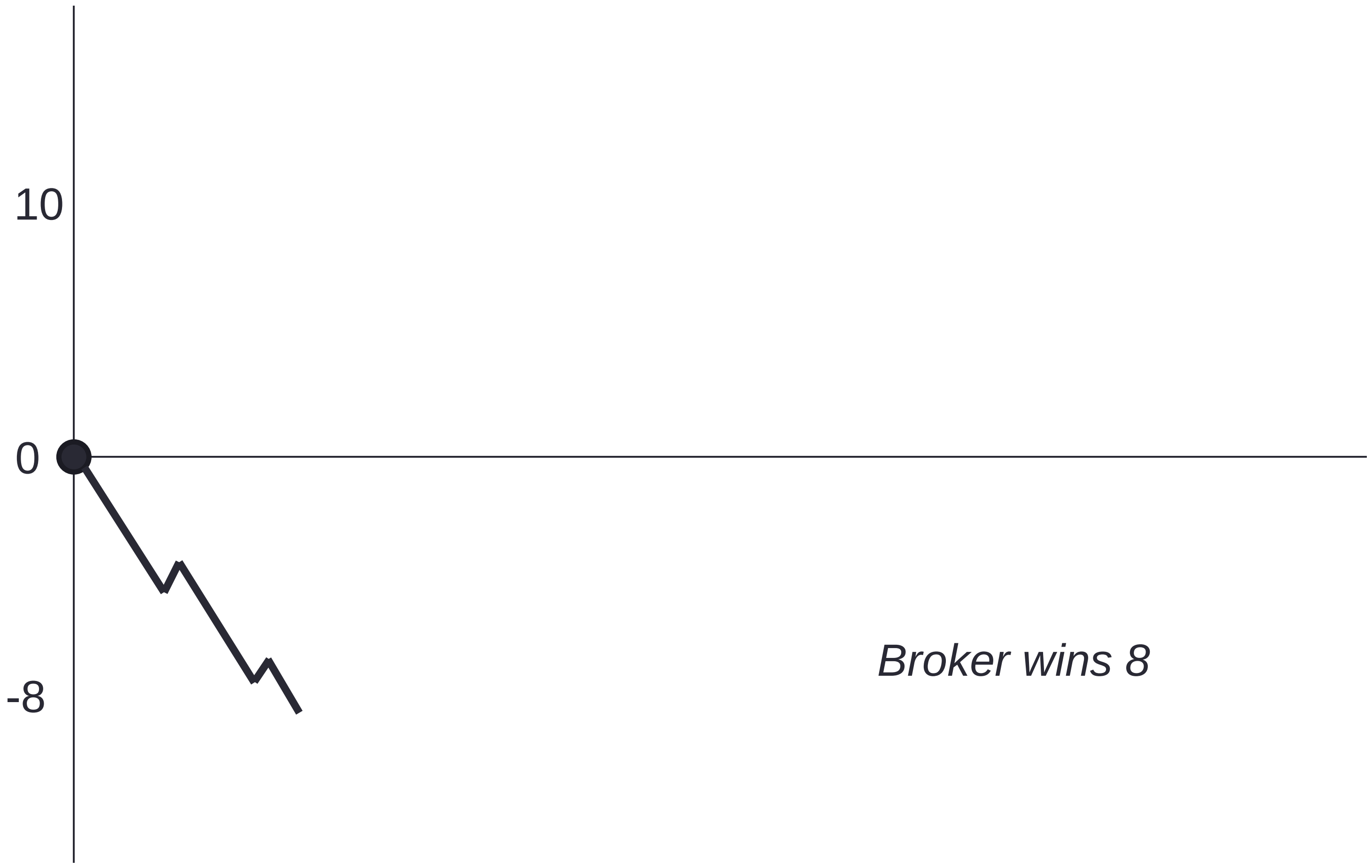
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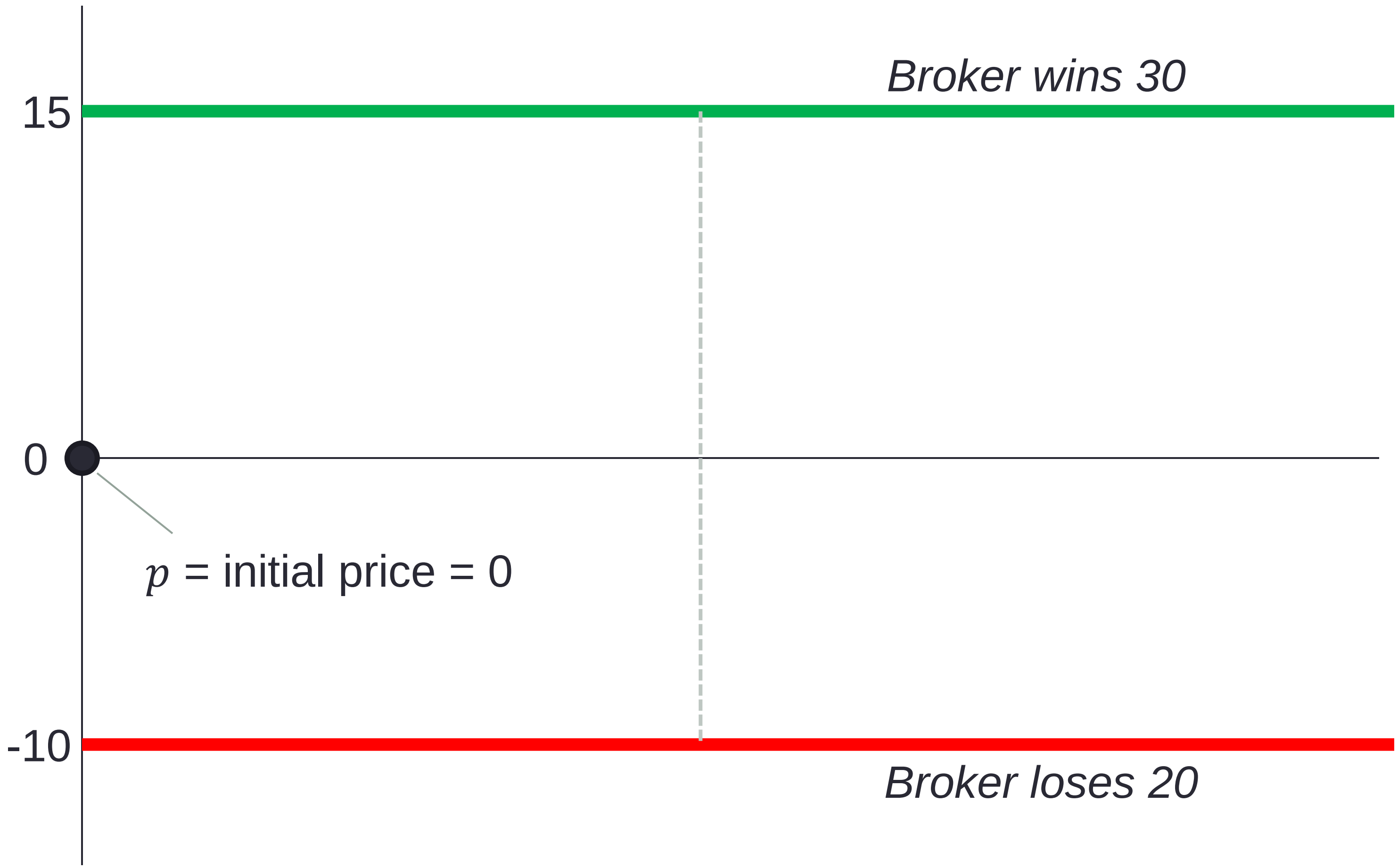
Broker wins 8


Broker wins 30
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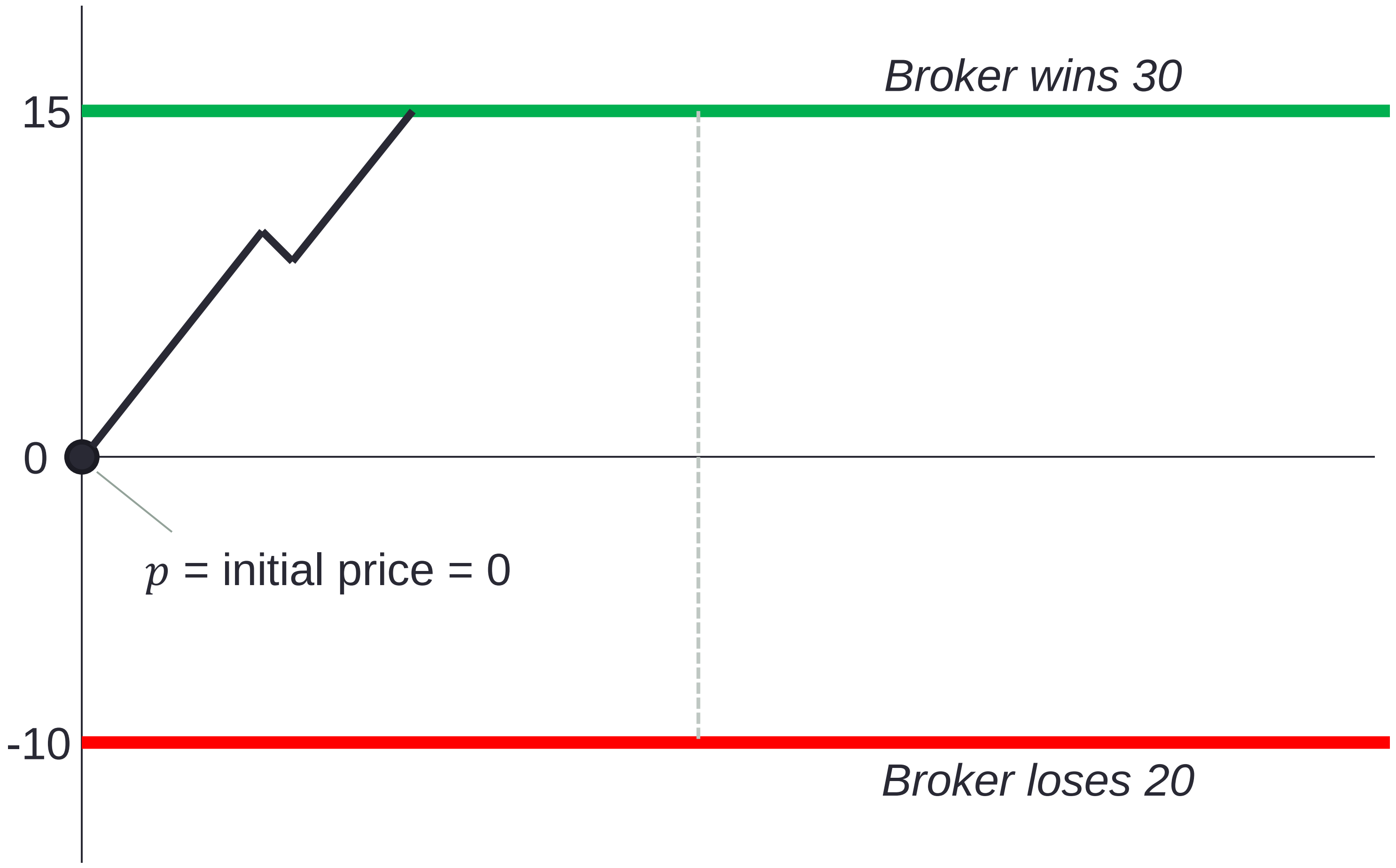
Broker loses 20



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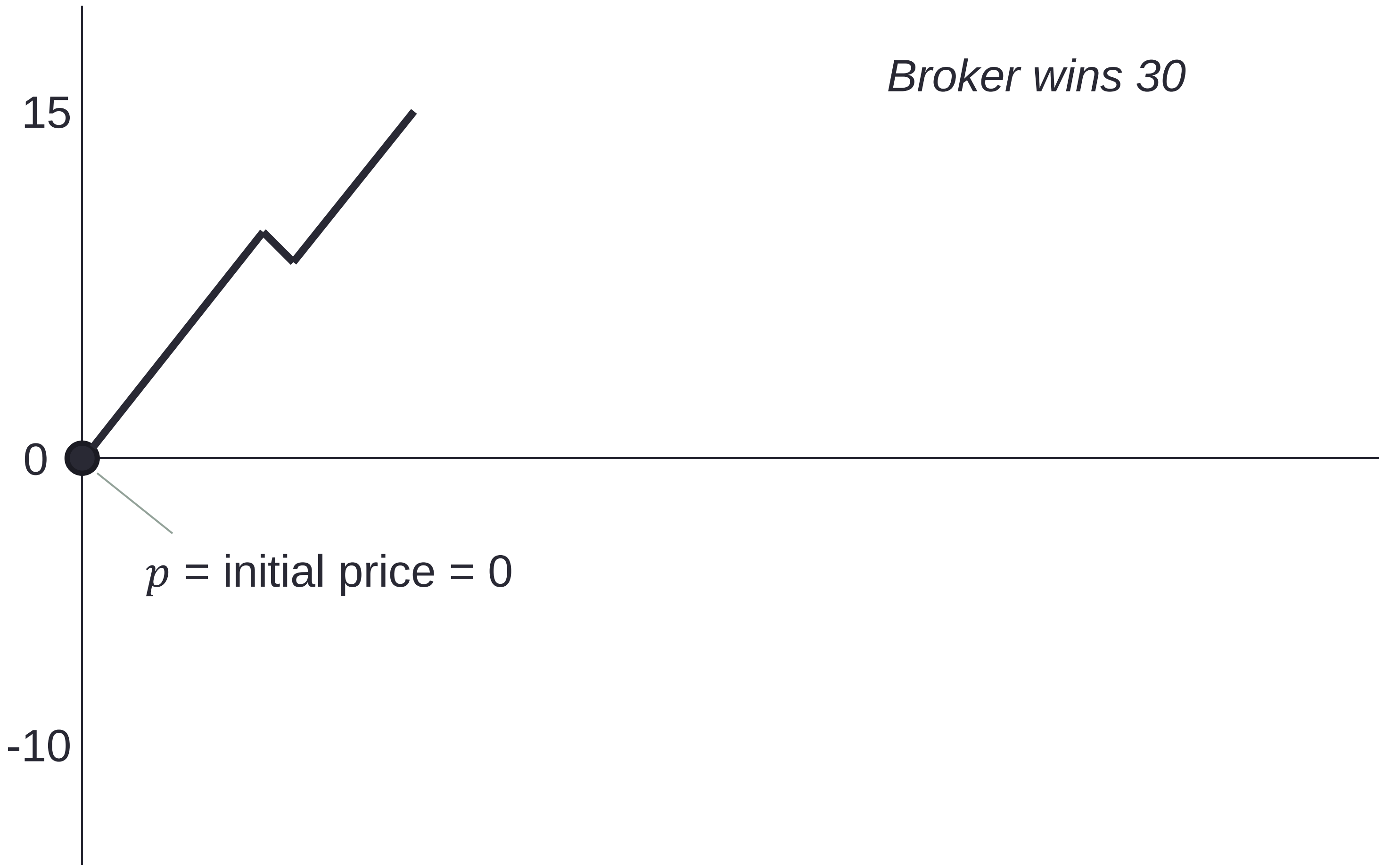
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The Maximum Trader Abuse problem

Trade = tuple (w, l, p_w, p_l)

- w = winning price p_w = profit at price w
- l = losing price p_l = profit at price l (negative)

Input: a set of trades T.
Goal: find a price movement M that maximizes total profit (must close every trade).

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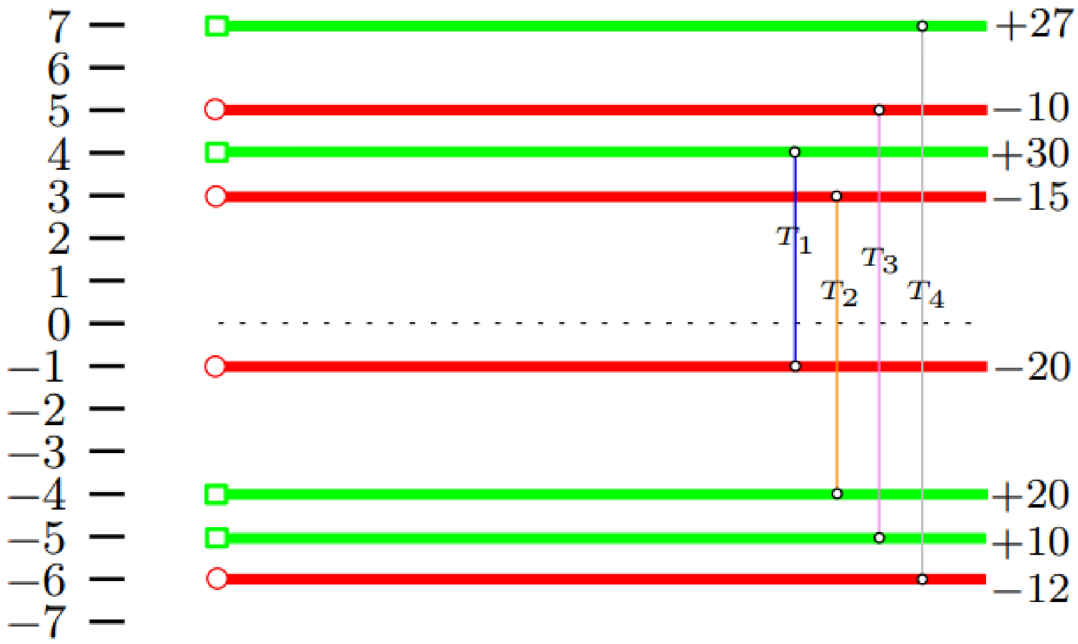
We study the Offline (15 mins) and Online (2 mins) setting.

The offline setting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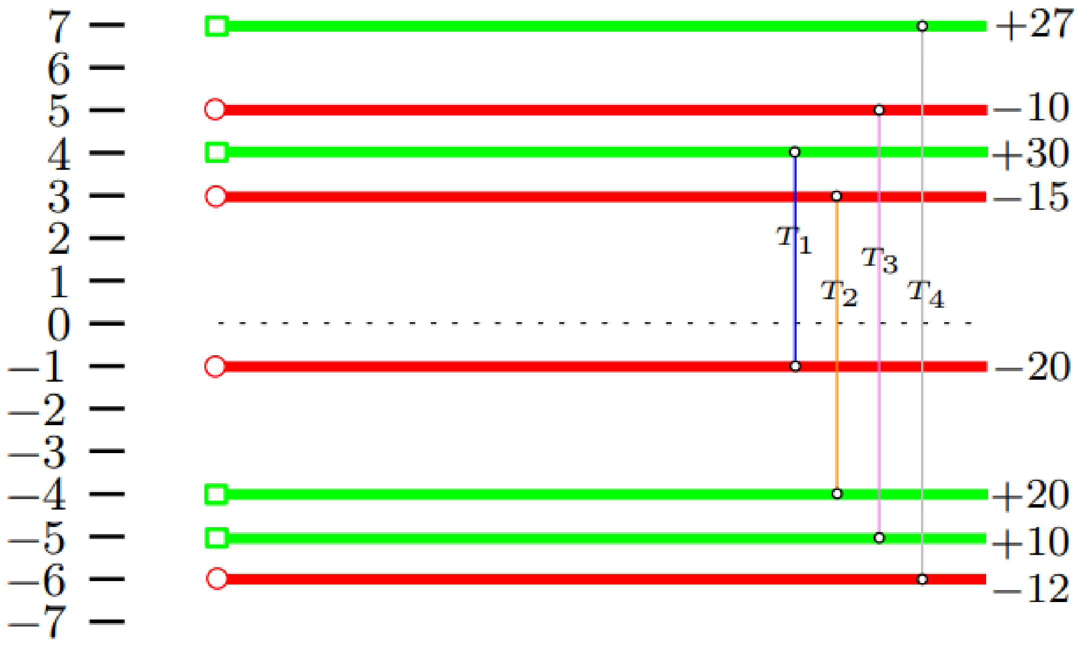
T_1
T_2
T_3
T_4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T_1
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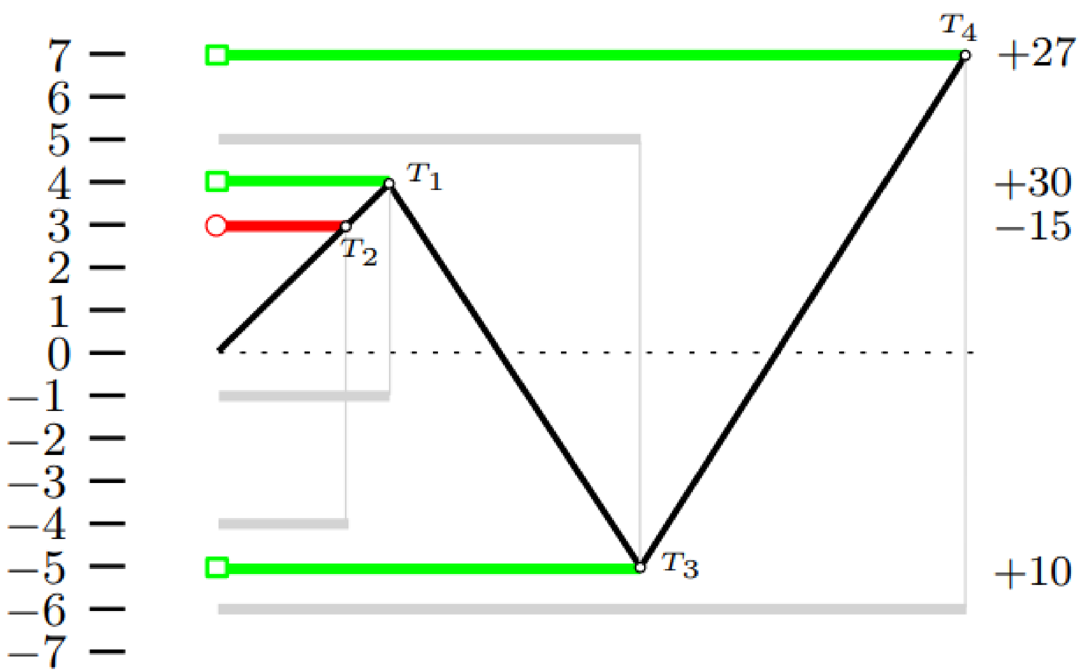
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T_1
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−15
T_2
T_3
+10

Compatible trades

Definition

Two trades T_1, T_2 are *compatible* if there exists a price movement that wins both trades. Otherwise, they are *incompatible*.

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Lemma

Two trades $T_1 = (w_1, l_1, p_w, p_l)$ and $T_2 = (w_2, l_2, q_w, q_l)$ are incompatible if and only if

- w_1 and w_2 have a different sign
- $|l_1| \leq |w_2|$ and $|l_2| \leq |w_1|$

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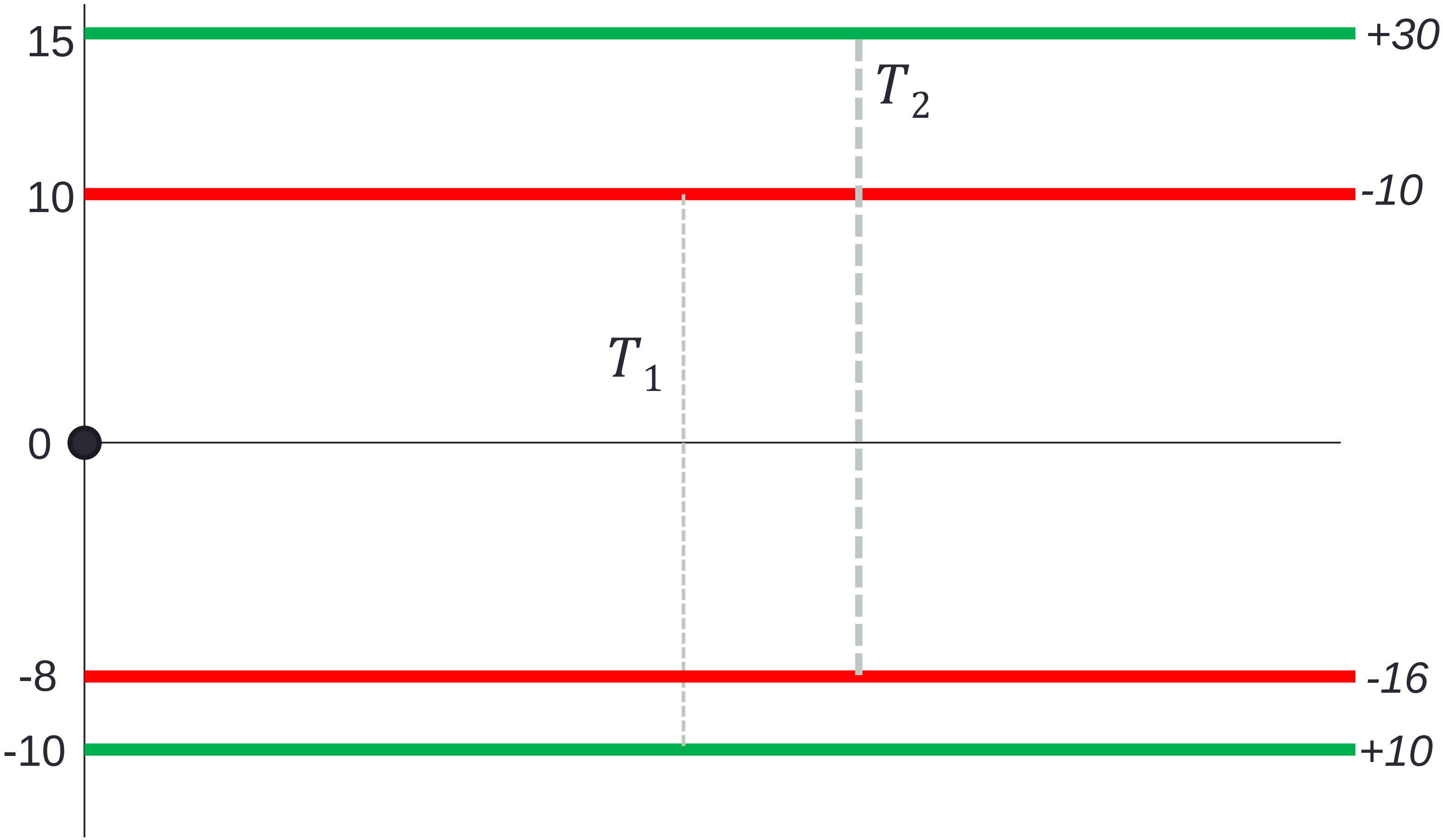
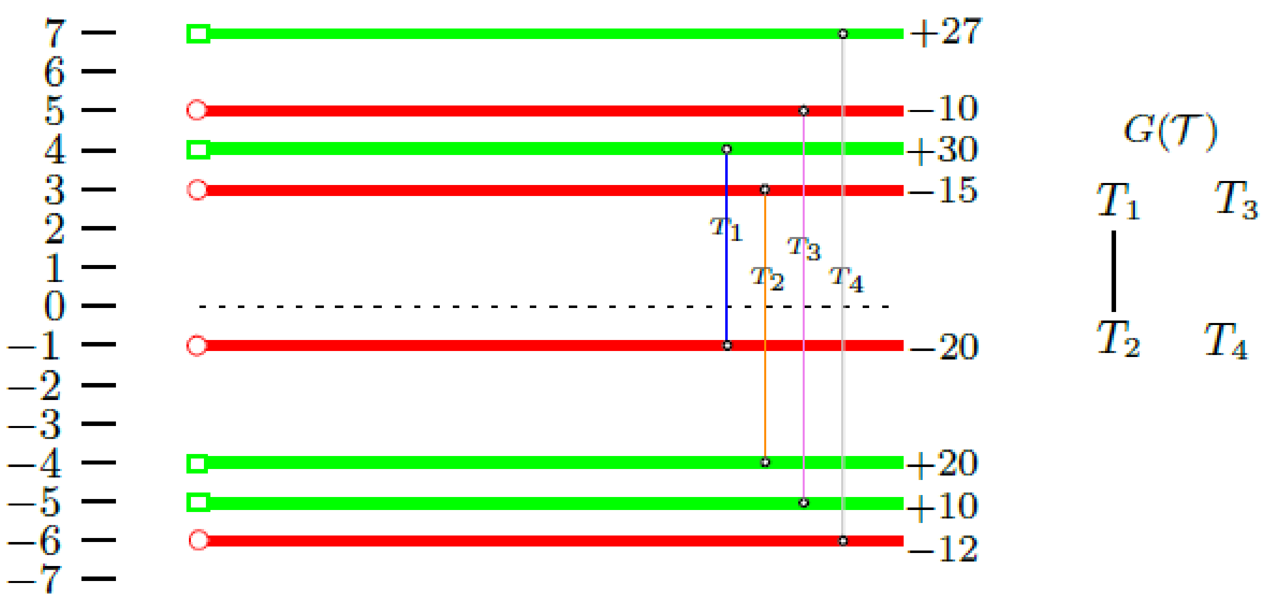


Definition

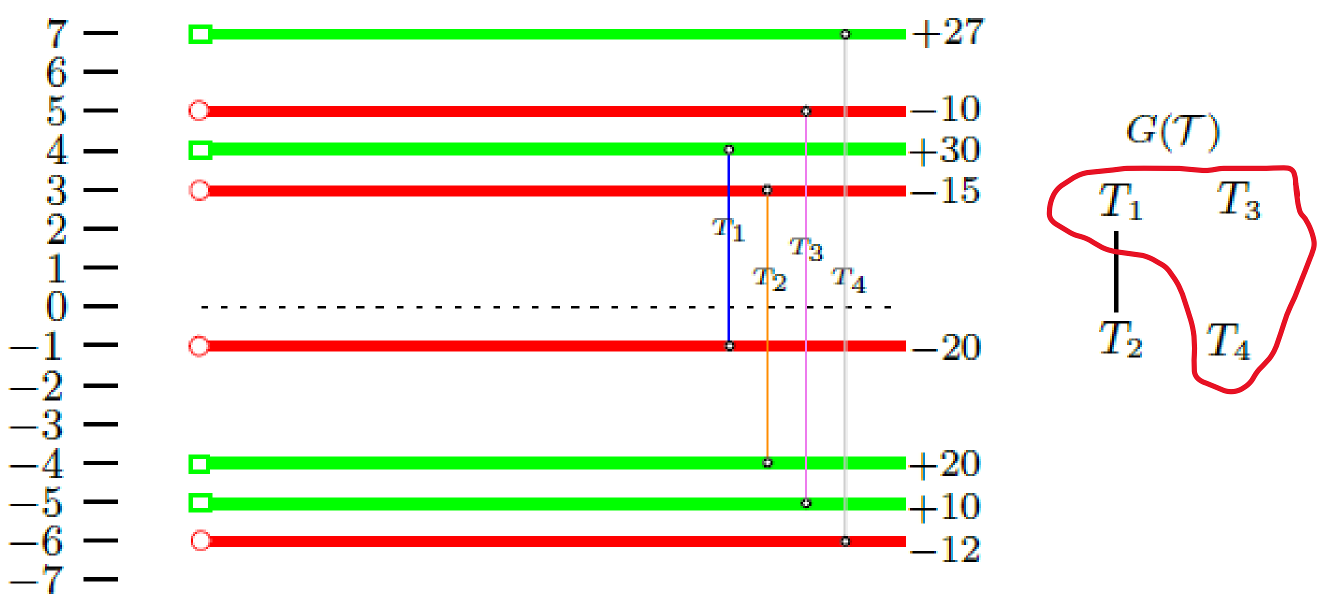
Given a set of trades T, the *trade conflict graph* $G(T)$ is the graph whose vertices are T, and T_1, T_2 share an edge iff they are incompatible.



Lemma

Consider $G(T)$ with vertex weights α where, for each trade $T_i = (w, l, p_w, p_l)$, we put $\alpha(T_i) = p_w - p_l$.

Then a maximum weight independent set of $G(T)$ corresponds to a set of trades that are won by an optimal price movement.



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Lemma

The trade conflict graph $G(T)$ is bipartite.

Proof: the trades won at price > 0 are compatible and thus form an independent set. Same with the trades won at price < 0.

Theorem

The maximum trader abuse problem reduces to finding a maximum weight independent set in a bipartite graph.

Solvable in time $O(n^3)$ or $O(nm)$ using flow techniques.

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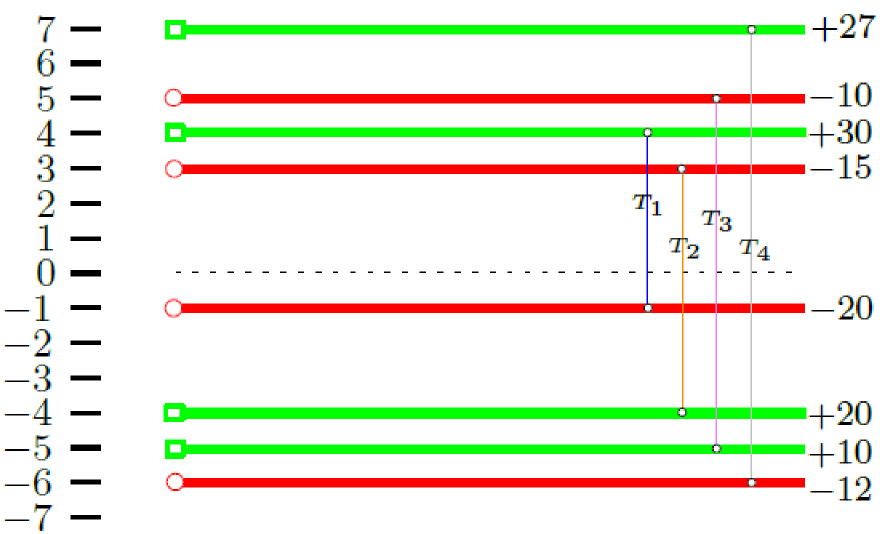
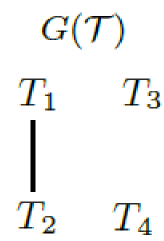
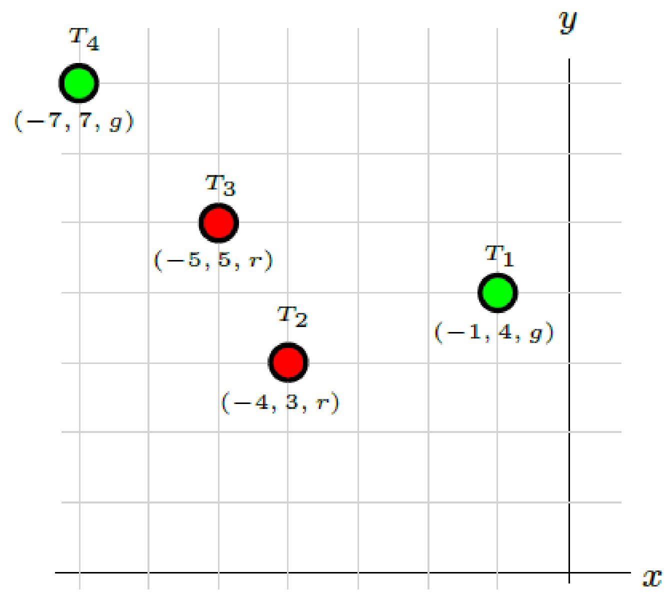
Question: can we characterize the trade conflict graph $G(T)$ to obtain something better? (and more interesting?)

Project each trade (w, l, p_w, p_l) as a point on the 2D plane. With coordinate x = min(w, l) and y = max(w, l).

Color the point green if $w > 0$ and red if $w < 0$.







Bicolored plane domination graphs

Colored point = triple (x, y, c)

- x, y are plane coordinates
- c is a color, either **red** or **green**

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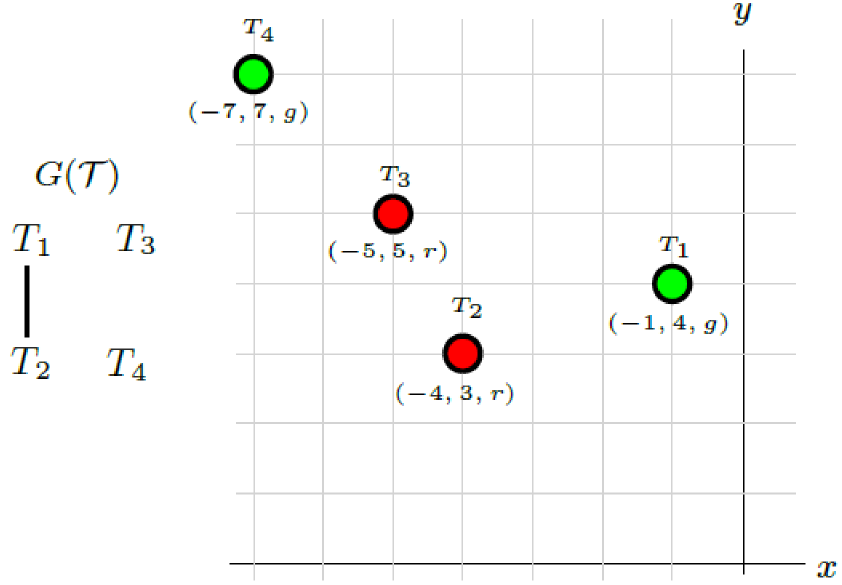
Colored point (x, y, c) dominates (x', y', c') if $x \geq x'$ and $y \geq y'$ (i.e. it's diagonally up-right)

Definition

A graph G is a bicolored plane domination graph if there exists a set of colored points P such that $V(G) = P$, and $(x, y, c), (x', y', c')$ share an edge if and only if $c \neq c'$ and the green point dominates the red point.



Theorem

A graph G is a trade conflict graph for some set of trades if and only if G is a bicolored plane domination graph.

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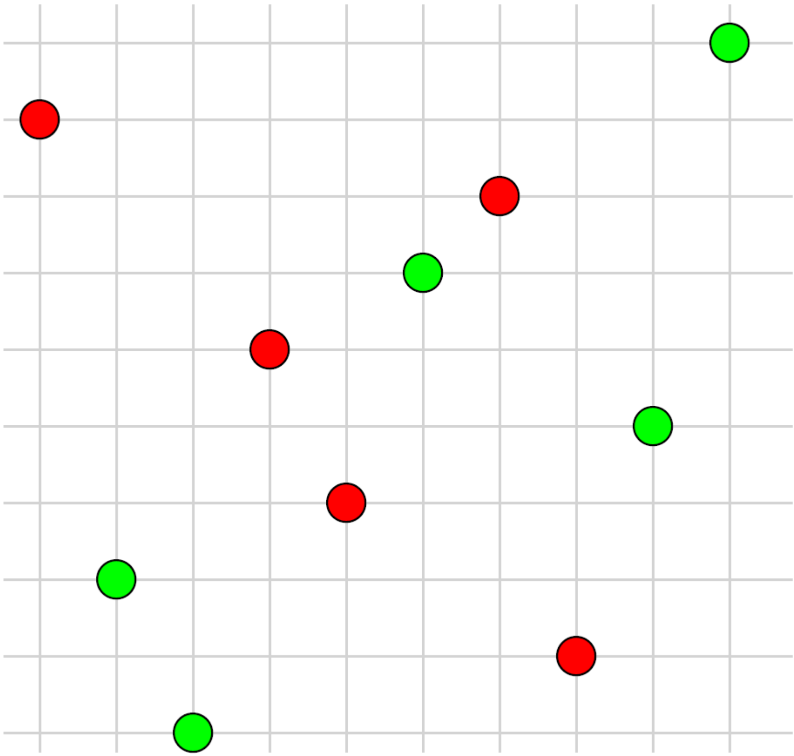
Useful to get an $O(n^2)$ time algorithm.

Lemma

Let G be a bicolored plane domination graph. Then G admits a colored point representation that has the ***permutation matrix property***, i.e. :

- each x, y coordinate is in $\{1, 2, \dots, n\}$
- each row has exactly one point, each column has exactly one point



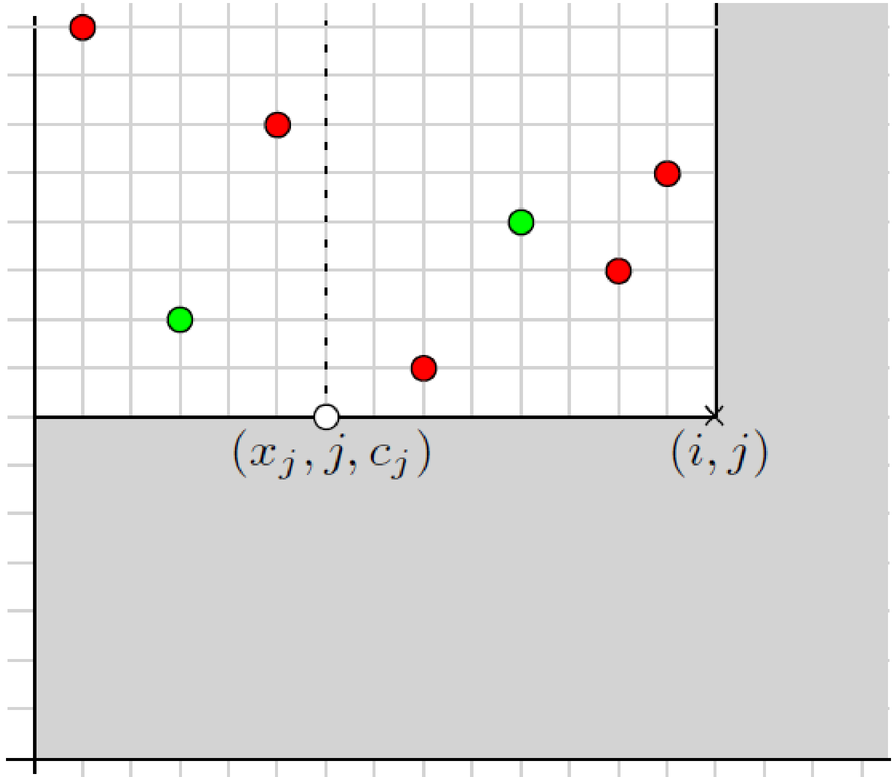
Max-weight independent set on bicolored plane domination graphs

Can be done in time $O(n^2)$

Dynamic programming on the permutation matrix grid representation.

Compute from bottom-right corner to top-left corner.

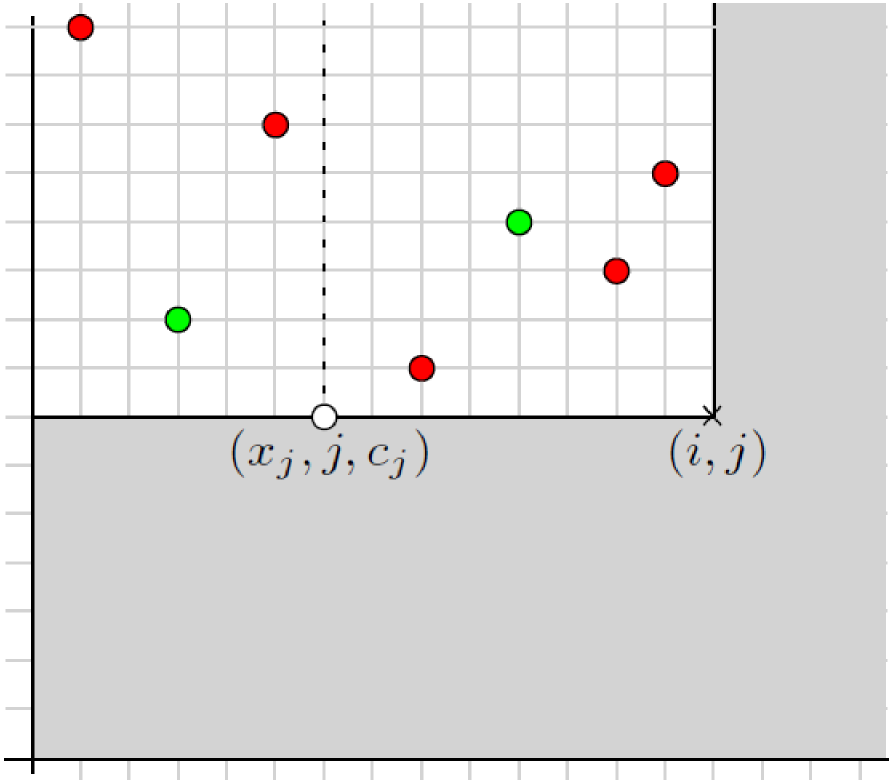
$I(i,j)$ = max weight indset of the subgrid from $(n,0)$ corner to (i,j)



$I(i,j)$ = max weight indset of the subgrid from $(n,0)$ corner to (i,j)

x_j = x coordinate of unique point on row j

$$I(i,j) = \begin{cases} I(i,j+1) & \text{if } x_j > i \\ h((x_j,j,c_j)) + I(i,j+1) & \text{if } x_j \leq i \text{ and } c_j \text{ is green} \\ \max(I(i,j+1), h(R(x_j,i,j)) + I(x_j-1,j+1)) & \text{if } x_j \leq i \text{ and } c_j \text{ is red} \end{cases}$$



Question

Given a bicolored plane domination graph, can a max-weight independent set be computed in $O(n)$ time?

The grid has only n points. We waste $O(n^2)$ on grid locations without points.

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Can we characterize trade conflict graphs, i.e. bicolored plane domination graphs?

Are they equivalent to some known graph class?

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Are they equivalent to some known graph class?

Paper says: they're chordal bipartite (bipartite + no cycle of lengths 6 or more).

Belief: somewhere between chordal bipartite and permutation graphs.

The online setting

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In reality, new trades can appear at any moment.

Traders can close their trades at any moment.

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Two-player game: broker and trader. Each turn:

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Rules:

- Broker's decisions are not based on the past events.
- If trade set is empty, price returns to 0.
- Trade profits must be linear.
  - For each trade, there is a d such that $profit = d * (open - close)$

There is only good broker strategy

Max potential strategy: on the broker's turn at price p, calculate:

- The profit $\$^+$ made if trader closed everything at price $p+1$
- The profit $\$^-$ made if trader closed everything at price $p-1$

If $\$^+ > \$^-$, move the price up, otherwise move the price down.

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- The profit $\$^{-}$ made if trader closed everything at price $p-1$

If $\$^{+} > \$^{-}$, move the price up, otherwise move the price down.

One of $\$^{+}$ or $\$^{-}$ is at least 0.

Broker can't lose money. If trader makes a mistake, positive profit is always achievable.

Theorem

If the broker uses any strategy other than always moving the price in the direction of maximum potential profit, then an optimal trader can make the broker bankrupt.

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Intuition : a suboptimal move from broker means negative potential profit. If that happens, trader closes everything at negative (broker) profit, and repeats the pattern infinitely.

▶ **Theorem 12.** *Suppose that the trader is restricted to linear online trades. Let $\mathcal{T}_i$ be a set of open trades at the start of the i-th turn, let p_i be the current price, and let $profit(i)$ be the total profit of the broker at the start of turn i. Then the following holds:*

1. *if $profit(i) + potent(\mathcal{T}_i, p_i) \geq 0$, then if the broker applies the maximum potential strategy from turn i and onwards, it achieves a total profit of at least $profit(i) + potent(\mathcal{T}_i, p_i)$ against any trader. Moreover, this is the maximum possible profit achieved against a trader with optimal play, passive or not;*
2. *if the broker does not move the price in the direction of maximum potential profit on turn i, then the broker makes a profit that is strictly less than $profit(i) + potent(\mathcal{T}_i, p_i)$ against a trader with optimal play, passive or not;*
3. *if $profit(i) + potent(\mathcal{T}_i, p_i) < 0$, then the broker incurs an infinite negative profit against a trader with optimal play, passive or not.*

Conclusion

- Finally, brokers can optimally abuse traders!

- Future
  - Extend trading model (randomness)
  - Improve algorithm
  - Characterize graph class

- THX